B.Sc Part I PAPER - I

Integral Calculus

Type II.

When the denominator consists of repeated and non - repeated linear factors such as $x-a_1$, $(x-a_2)^2$, etc.,
then the given fraction can be expressed as

$$\frac{A}{(x-a_1)} + \frac{B}{(x-a_2)} + \frac{C}{(x-a_3)^2} + \cdots$$

where A, B, C are constants to be evaluated and finally the integration can be carried out

The following example illustrates the process fully.

Integrate $\displaystyle\int \frac{dx}{(x-a)^2(x-b)(x-c)}$

$$= \frac{A}{(x-a)} + \frac{B}{(x-a)^2} + \frac{C}{(x-b)} + \frac{D}{(x-c)}$$

$$\therefore 1 = A(x-a)(x-b)(x-c) + B(x-b)(x-c) + C(x-a)^2(x-c) + D(x-a)^2(x-b) \quad \text{— (1)}$$

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Putting $x=a, b, c$ in (1) successively we get

$B = \dfrac{1}{(a-b)(a-c)}$, $C = \dfrac{1}{(b-a)^2(b-c)}$

$D = \dfrac{1}{(c-a)^2(c-a)}$

To get A, equate the coefficients of x^3 viz

$0 = A+C+D = A + \dfrac{1}{(b-a)^2(b-c)} + \dfrac{1}{(c-a)^2(c-b)}$

$\therefore \displaystyle\int \frac{dx}{(x-a)(x-b)(x-c)} = \frac{b+c-2a}{(b-a)^2(c-a)^2} \log(x-a)$

$+ \dfrac{1}{(a-b)(c-a)} \cdot \dfrac{1}{(x-a)} + \dfrac{1}{(b-a)^2(b-c)} \log(x-b)$

$+ \dfrac{1}{(c-a)^2(c-b)} \log(x-c) + K$